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FREE VIBRATION OF COMPOSITE LAMINATED PLATES SUBJECTED TO HYGROTHERMAL LOADS

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ABSTRACT
Carbon fibre reinforced polymers (CFRP) are now used extensively in modern aircraft structures. Their introduction occurred because of their better specific stiffness and strength compared with conventional aerospace materials such as Al alloys. In recent years, these materials have been used extensively in advanced V/STOL aircraft such as McAIr AV-8B/GR MK5. These materials have been used extensively in advanced aircraft. In such cases certain areas of the aircraft structure may be subjected to thermal and moisture loading. It is therefore necessary to predict the free vibration response of the composite plate when subjected to thermal and moisture loadings. First-order shear deformation theory was applied in Finite Element Method (FEM) to free vibration analysis of fully clamped laminated plate. A theoretical equation was developed and compared it with FEM result. The layup investigated were $[0_2 , 90_2]$, $[0 ,90]_4$, $[\pm45_2]$, $[\pm45]_4$, $[0, 90]_{2s}$, $[\pm45]_{2s}$, $[0, \pm45, 90]_s$ for 2% and 4% moisture content with uniform temperature of 20°C, 40°C, 60°C, 80°C, 100°C, 120°C. Natural frequencies are decreased with temperature and more significant at 4% moisture uptake.

Keywords*:* Natural Frequency, CFRP, FSDT, Hygrothermal.

1. INTRODUCTION

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc.

In many cases, using composites is more efficient. For example, in the highly competitive airline market, one is continuously looking for ways to lower the overall mass of the aircraft without decreasing the stiffness and strength of its components. This is possible by replacing conventional metal alloys with composite materials. Even if the composite material costs may be higher, the reduction in the number of parts in an assembly and the savings in fuel costs make them more profitable

Composite plate of Carbon fibre reinforced polymer (CFRP) is now used extensively in modern aircraft structures. In recent years, these materials have been used extensively in advanced V/STOL aircraft such as McAIr AV-8B/GR MK5 [1]. In such cases certain areas of the aircraft structure may be subjected to thermal and moisture loading. It is therefore necessary to predict the free vibration response of the composite plate when subjected to thermal and moisture loadings. To reveal the thermal and moisture effect on the vibration of a plate with uniform temperature change, it is either that the mechanical properties of the plate are considered as functions of temperature [2-3] or, the nonlinear plate theories, or the nonlinear strains should be applied [4-5]. The dynamic response of simply supported and clamped CFRP composite plate subjected to thermal and moisture environment is carried out using the general purpose finite element program ANSYS.

2. COMPOSITE LAMINATED PLATE THEORIES

There are several theories currently in use for the composite laminated plates [Reddy (1997)]. The simplest one the classical laminated plate theory (CLPT). Composites have a very low transverse shear modulus compared to their in-plane moduli. Therefore, the CLPT may not be sufficient for the dynamic analysis of composite plates. To account for the shear deformation, the first order shear deformation theory (FSDT) can be adopted for the analysis. The FSDT assumes that the transverse normals are straight but not perpendicular to the mid surface after deformation. Thus transverse shear strains are constant through the cross section. Since the actual shear stress is not constant, FSDT uses a shear correction factor. In FEM analysis FSDT is used.

For formulating theoretical equation consider a rectangular plate with dimensions L_x and L_y simply supported along its four edges. The layup of the plate is symmetrical, [B] = 0. The mass of the plate is uniform

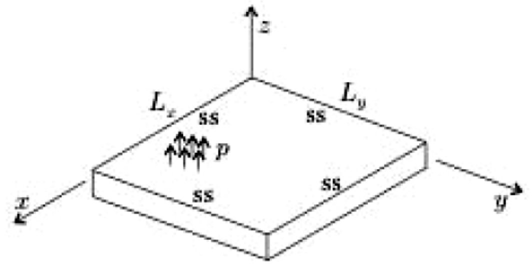

Fig 1. Rectangular Simply Supported Plate

For a linear elastic system the strain energy of volume V is defined as

$$U=\frac{1}{2}\iiint\left(\varepsilon_x\ \sigma_x+\varepsilon_y\sigma_y+\varepsilon_z\ \sigma_z+\gamma_{yz}\tau_{yz}+\gamma_{xz}\tau_{xz}+\gamma_{xy}\tau_{xy}\right)dV \qquad (1)$$

For plane stress assumption strain energy simplifies to

$$U=\frac{1}{2}\int_0^{L_x}\int_0^{L_y}\int_{h_b}^{h_t}\left(\varepsilon_x\ \sigma_x+\varepsilon_y\sigma_y+\gamma_{xy}\tau_{xy}\right)dzdydx \qquad (2)$$

For a simply supported plate (symmetrical layup) subjected to out-of-plane loads only, the in plane strains in the midplane are zero.

$$U=\frac{1}{2}\int_0^{L_x}\int_0^{L_y}\left(\kappa_x\ \ \kappa_y\ \ \kappa_{xy}\right)\begin{matrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66}\end{matrix}\ \begin{matrix}\kappa_x \\ \kappa_y \\ \kappa_{xy}\end{matrix}\ dydx$$

(3)

When a plate undergoes free, undamped vibration the deflection of the plate is sinusoidal with respect to time t

$$w^o=\overline{w}^o\mathrm{Sin}\left(\omega t\right)=\overline{w}^o\mathrm{Sin}\left(2\pi ft\right) \qquad (4)$$

The relationship between curvature and the deflections

$$\begin{matrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{matrix}=\begin{matrix}\frac{\partial u_o}{\partial x} \\ \frac{\partial v_o}{\partial y} \\ \frac{\partial u_o}{\partial y}+\frac{\partial v_o}{\partial x}\end{matrix}+Z\ \begin{matrix}\frac{\partial^2 w_o}{\partial x^2} \\ \frac{\partial^2 w_o}{\partial y^2} \\ 2\frac{\partial^2 w_o}{\partial x\partial y}\end{matrix}$$

(5)

By using equation (4) we get

$$\overline{U}=\frac{1}{2}\int_0^{Lx}\int_0^{Ly}[\ D_{11}(\frac{\partial^2 w_o}{\partial x^2})^2+D_{22}(\frac{\partial^2 w_o}{\partial y^2})^2+D_{66}(\frac{2\partial^2 w_o}{\partial x\ \partial y})^2$$
$$+2(D_{12}\frac{\partial^2 w_o}{\partial x^2}\frac{\partial^2 w_o}{\partial y^2}+D_{16}\frac{\partial^2 w_o}{\partial x^2}\frac{2\partial^2 w_o}{\partial x\partial y}$$
$$+D_{26}\frac{\partial^2 w_o}{\partial y^2}\frac{2\partial^2 w_o}{\partial x\partial y})]dydx \qquad (6)$$

Following whitney [6], we obtain the natural frequencies of this plate by the energy method. By introducing w_0 into the expression for U we obtain

$$U=\overline{U}\ \ \mathrm{Sin}^2(2\pi ft) \qquad (7)$$

p is the lateral force (per unit area) acting on the plate. In the case of a freely vibrating plate p is the inertia force.

$$p=\ \rho\frac{\partial^2 w^o}{\partial t^2}=\rho(2\pi f)^2 w^o\mathrm{Sin}\left(2\pi ft\right) \qquad (8)$$

The kinetic energy of the plate is

$$K=\frac{1}{2}\int_0^{L_x}\int_0^{L_y}\rho(\frac{dw^o}{dt})^2\,dydx. \qquad (9)$$

Substituting of the deflection into this equation yields

$$K=\frac{1}{2}(2\pi f)^2\cos^2(2\pi ft)\int_0^{L_x}\int_0^{L_y}\rho\overline{w_o}^2 dydx \qquad (10)$$

According to the law of conservation of energy the change in strain energy from time t=0 to time t equals the change in kinetic energy during this time

$$\left(U_t\quad U_{t=0}\right)=\ \left(K_t\quad K_{t=0}\right)$$

Initially, at time t=0 the strain energy is zero equation of U but at time $t=\frac{1}{4f}$ the kinetic energy is zero, thus we have

$$U_{t=\frac{1}{4f}}=K_{t=0} \qquad (11)$$

Now using from equation (7) and (8) into equation (9)

$$\frac{1}{2}(2\pi f)^2\int_0^{L_x}\int_0^{L_y}\rho\overline{w}_o{}^2 dydx=\overline{U} \qquad (12)$$

For the simply supported plate under consideration the geometrical boundary conditions require the deflections by zero along the edge.

$$w^o=0\quad \text{at}\quad \begin{matrix} x=0 & \text{and} & 0\leqslant y\leqslant L_y \\ x=L_x & \text{and} & 0\leqslant y\leqslant L_y \\ 0\leqslant x\leqslant L_x & \text{and} & y=0 \\ 0\leqslant x\leqslant L_x & \text{and} & y=L_y\end{matrix} \qquad (13)$$

The following deflection satisfies these geometrical boundary conditions

$$w^o = \overline{w}^o \sin(2\pi f t)$$

Where $\overline{w}^o$ is

$$\overline{w^o} = \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} \sin\frac{i\pi x}{L_x} \sin\frac{i\pi y}{L_y}, \quad (14)$$

Where I and J are the number of terms, chosen arbitrarily, for the summations and w_{ij} are constants. According to the Rayleigh principle the frequency of vibration of a conservative system has a minimum value in the neighborhood of the fundamental mode. We express this principle in the form

$$\frac{\partial f}{\partial w_{ij}} = 0, \quad (15)$$

By substituting equation (13) into equation (14) the natural frequencies are calculated as follows

$$f_{ij} = \frac{1}{\pi}\sqrt{\frac{\lambda_{ij}}{\rho L_x L_y}} \quad (16)$$

Using Rayleigh's energy method [7] the natural frequencies of an orthotropic plate is given by following equation

$$f_{ij} = \sqrt{\frac{\pi^2}{4\rho}\left\{D_{11}\left(\frac{i}{L_x}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{i}{L_x}\right)^2\left(\frac{j}{L_y}\right)^2 + D_{22}\left(\frac{j}{L_y}\right)^4\right\}} \quad (17)$$

Natural frequencies of CFRP plates are measured by ANSYS and compared with results from Equation (15). During FEM formulation to calculate natural frequency following equation is used

$$[K]\{\phi_i\} = \omega_i^2 [M]\{\phi_i\} \quad (18)$$

Where [K] is stiffness matrix, $\{\phi_i\}$ is mode shape vector of mode i, ω_i is the natural circular frequency, ω_i^2 is the eigen value and [M] is the mass matrix.

3. FINITE ELEMENT MODELING

The finite element program ANSYS is used to study the thermal effects on the dynamic properties of CFRP plates under simply supported and clamped boundary conditions. An eight node layered shell element (Shell 99) is used to model the layered plate. The Shell99 is an eight nodded linear layered structural shell element that can take maximum of 250 equal thickness layers. This element has six degree of freedom at each node; three translation x,y,z and three rotation about the x,y,z axes. The formulation of the elements includes first order shear deformation plate theory

4. HYGROTHERMAL ANALYSIS

Moisture uptake concentration is defined in this paper as percentage uptake by weight. This may calculated using following equation

Moisture Uptake concentration % = $\frac{Weight(final) - Weight(initial)}{Weight(initial)}$

In most cases we are considering common aerospace epoxy resins and fibers in laminate form. This means that the diffusion will be primarily through the laminate faces and only a small effect. Moisture absorbed by the laminates depends on time

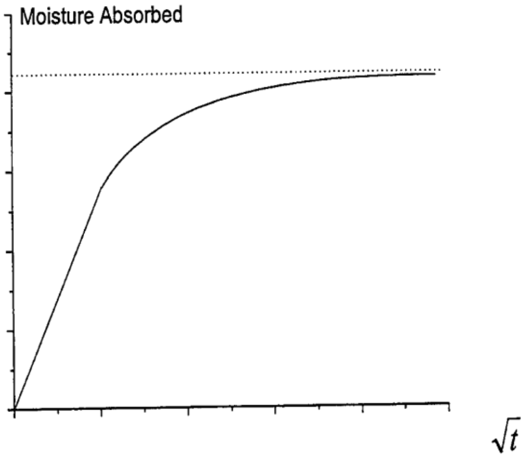

Fig 2. Moisture Uptake versus root time under constant Humidity and temperature conditions

It has been shown that for many composite materials that the maximum moisture uptake is related to humidity by equation $M_m = a\text{ø}^b$, where ø is relative humidity, and *a* and *b* are constants. Following figure shows maximum moisture uptake of boron epoxy materials.

The term moisture uptake, moisture absorbed, moisture concentration and weight gain are used interchangeably in the literature. All terms refer to moisture uptake of the resin or composite by weight.

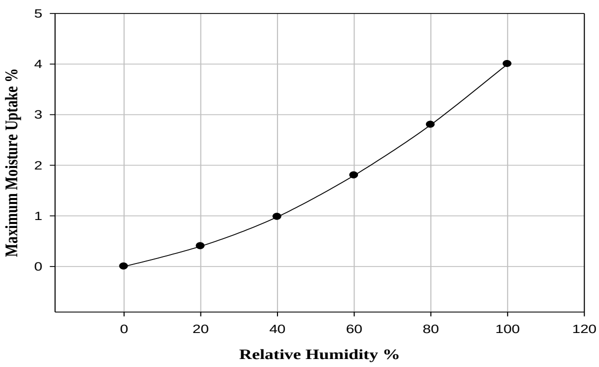

Fig 3. Maximum Moisture Uptake versus Relative Humidity % at constant temperature conditions.

So, maximum moisture uptake at 100% humidity is 4%. As a result we take the effect of natural frequency at 2% and 4 % moisture uptake for this study.

According to Chamis [9], the relationship between the wet resin and dry resin mechanical properties can be stated as

$$\frac{P_{HTM}}{P_o} = \left(\frac{T_{gwr} \quad T}{T_{gdr} \quad T_o}\right)^{0.5} \qquad (17)$$

Where *P* is the property to be measured, HTM stands for hygrothermal mechanical, T_{gwr} is glass transition temperature of wet resin and T_{gdr} is the glass transition temperature of dry resin. *T* is the temperature at which property to be measured and T_o is the room temperature. This is a steady state equation that relates the properties of the matrix for dry and wet conditions at particular values of moisture content and temperature.

The relation between T_{gwr} and T_{gdr} in terms of moisture content is given by Chamis and is stated as

$$T_{gwr} = (0.005\ m2 - 0.1m + 1.0)\ T_{gdr} \qquad (18)$$

where *m* is the moisture content expressed in weight percent ($m \leq 10\%$)

4 RESULTS AND DISCUSSION

At first, the present FE model has been validated by comparing its results with those of Equation (17) for simply supported $[0, \pm45, 90]_s$ composite laminated plate. Then study involves a parameter study of the free vibration response, by using the finite element method, of a number of angle-ply and crossed ply clamped (symmetric and unsymmetric) laminated plates. The plates are subjected to uniform moisture gain. The layup investigated are $[0_2, 90_2]$, $[0, 90]_4$, $[\pm45_2]$, $[\pm45]_4$, $[0, 90]_{2s}$, $[\pm45]_{2s}$, $[0, \pm45, 90]_s$ which denoted as Plate 1,Plate 2,Plate 3,Plate 4, Plate 5, Plate 6, Plate 7 respectively. The material properties used in these laminated plates are typically values measured for XAS/914c [8]. The frequencies ω are nondimensionalized according to

$$\omega_d = \omega a^2 (\rho/E_2)^{0.5} / h$$

Where,

ω_d = Nondimensional frequency

a = 0.3 m

ρ = 1630 kg/ m3

E_2 = 8.5 GPa

We know that Increase in temperature and moisture will cause a decrease in the matrix properties and thus reduction in the transverse Young's Modulus (E_2) and the shear modulus (G_{12}) As a result with increasing temperature and moisture fundamental frequencies of laminated composite plates are decreased as shown in Figure 4. This effect becomes more significant as the temperature approaches the glass transition of the matrix.

The values of non dimensional fundamental frequencies are lower for Single cross ply, [02, 902] and angle ply [±452]. Natural frequency for Single cross ply, [02, 902] and angle ply [±452] is reduced at a higher rate than any other composite laminated plates. O

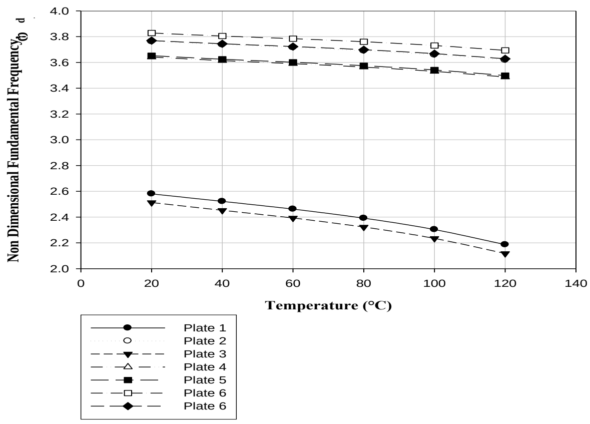

Fig 4. Variation of Nondimensional Fundamental Frequencies (**ω_d**) with change of Temperature for composite plate of different layup at 2% moisture uptake.

Slight variations of fundamental frequency are predicted for the multi-ply laminates [0, 90] 2s, [0, ±45, 90] with temperature.

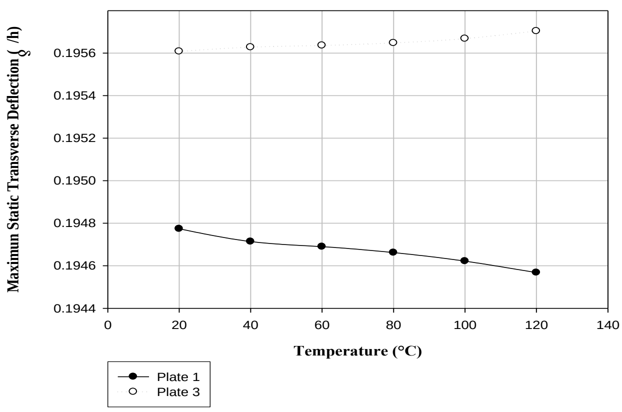

Fig 5. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup at 2% moisture uptake

Figures 5 and 6 show the maximum static transverse deflection with temperature. The value of maximum static transverse deflection is higher for Single cross ply, [02, 902] and angle ply [±452] and minimum for multiply [0, 90]4, [0, 90]2s composites.

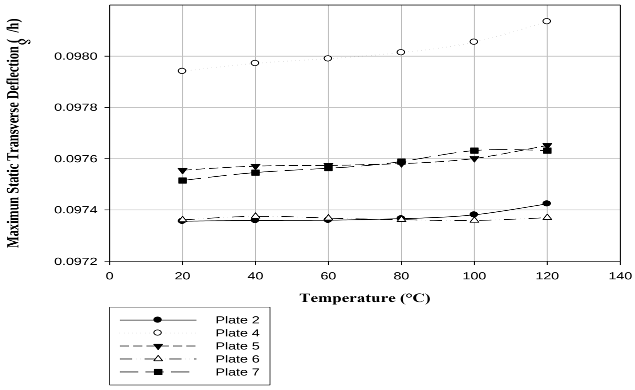

Fig 6. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup at 2% moisture uptake.

The value of maximum static transverse deflection is increased with temperature for exception of [02, 902] composite layup and this value is higher at 120°C.

In Figure 7 it is seen that with increasing temperature and moisture fundamental frequencies of laminated composite plates are decreased. This effect becomes more significant as the temperature approaches the glass transition of the matrix. At 4% moisture uptake and at 120°C the glass transition temperature is near about 119°C. Glass transition temperature is closed to the exposed temperature of 120°C as a result natural frequencies are decreased drastically and also static transverse deflections are increased suddenly.

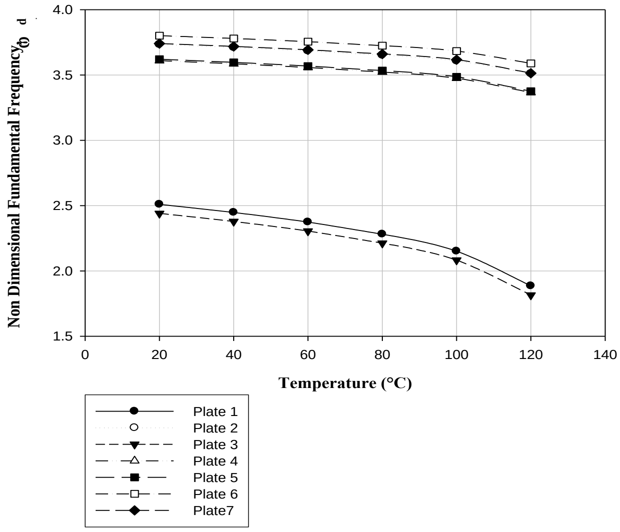

Fig 7. Variation of Nondimensional Fundamental Frequencies (ω_d) with change of Temperature for composite plate of different layup at 4% moisture uptake.

Figures 8, 9 show the variation of maximum static transverse deflection with temperature. The maximum transverse deflection with temperature for 4% moisture uptake is similar to 2% moisture uptake. The value of maximum transverse deflection is increased at 120°C suddenly at 4% moisture uptake as the glass transition temperature is near about the exposed temperature. The variation of maximum static transverse deflection is not significant between 20°C to 80°C.

Figure 10 shows the comparison between the theoretical results and finite element results of [0, ±45, 90]$_s$ Laminate. In both cases boundary condition is simply supported. It can be seen that the finite element result are in good agreement with the theoretical values.

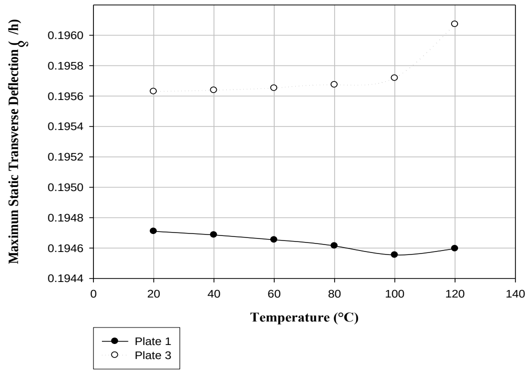

Fig 8. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup at 4% moisture uptake.

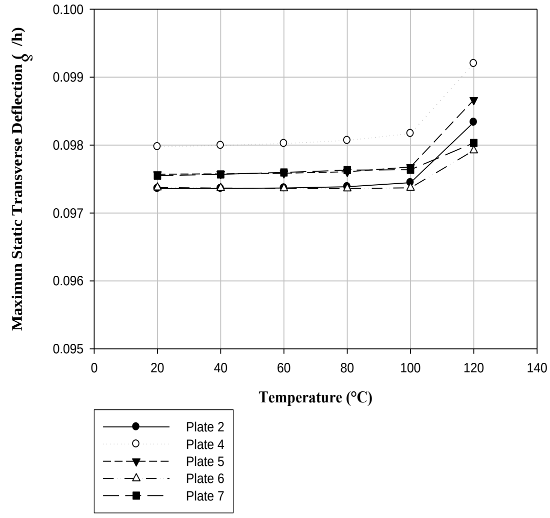

Fig 9. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup at 4% moisture uptake.

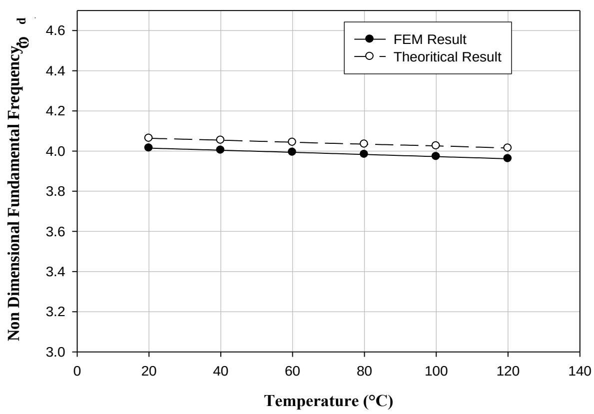

Fig 10. Comparison of Nondimensional Fundamental Frequencies (ω_d) for composite plate with theoretical Values.

Table 1: Fundamental Nondimensional Frequency for Various Laminated Square Clamped Plates (At 2% Moisture Uptake)

Composite Layup	20°C	40°C	60°C	80°C	100°C	120°C
$[0_2,90_2]$	2.58	2.52	2.46	2.39	2.30	2.19
$[0,90]_4$	3.76	3.74	3.72	3.69	3.66	3.62
$[\pm 45]_2$	2.51	2.45	2.39	2.32	2.23	2.12
$[\pm 45]_4$	3.64	3.61	3.59	3.56	3.53	3.49
$[\pm 45]_{2s}$	3.65	3.62	3.60	3.57	3.54	3.50
$[0,90]_{2s}$	3.83	3.80	3.78	3.76	3.73	3.69
$[0,\pm 45,90]_s$	3.77	3.74	3.72	3.70	3.67	3.63

Table 2: Fundamental Nondimensional Frequency for Various Laminated Square Clamped Plates (At 4% Moisture Uptake)

Composite	20°C	40°C	60°C	80°C	100°C	120°C
$[0_2,90_2]$	2.51	2.45	2.37	2.28	2.15	1.89
$[0,90]_4$	3.73	3.71	3.68	3.65	3.61	3.51
$[\pm 45]_2$	2.44	2.38	2.30	2.21	2.08	1.81
$[\pm 45]_4$	3.61	3.58	3.56	3.52	3.47	3.36
$[\pm 45]_{2s}$	3.62	3.60	3.57	3.53	3.49	3.38
$[0,90]_{2s}$	3.80	3.78	3.75	3.72	3.68	3.59
[0,±45,90	3.74	3.72	3.69	3.66	3.62	3.51

5. CONCLUSIONS

A parametric study was carried out with a number of cross-ply and angle ply square clamped laminates to observe the effect of uniform moisture content on the free vibration characteristics of laminates. The parametric study showed that the single cross ply $[0_2, 90_2]$ and double angle ply $[\pm 45_2]$ laminates showed greatest variations in natural frequencies. For 2% and 4% moisture concentration, fundamental frequency is decreased 15% and 26% from 20°C to 120°C of composite plate Single cross ply, $[0_2, 90_2]$ and Angle ply $[\pm 45_2]$ respectively Only slight variations were predicted for the multi-ply laminates $[0, 90]_4$, $[\pm 45]_4$, $[0, 90]_{2s}$, $[\pm 45]_{2s}$, [0, ±45, 90]s.

6. ACKNOWLEDGEMENTS

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